Geodetic Deformation Analysis with respect to Observation Imprecision

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TS 68 – Deformation Measurements of Dams

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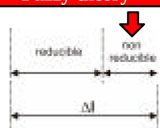
Motivation

Focus in this presentation :

- Measurement process:
  - Stochasticity → **Stochastics (Bayesian approach)**
  - Imprecision → **Interval mathematics Fuzzy theory**
  - (Outliers)

GUM

Guide to the Expression of Uncertainty in Measurement



Systematic effects

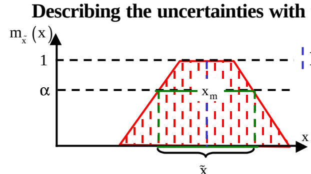
Motivation

Requirements:

➢Adequate description of Stochastics ✓

➢**Adequate description of Imprecision**

Describing the uncertainties with fuzzy intervals:



Precise stochastic midpoint x_m

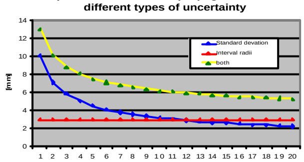
Imprecision

Motivation

Mean value $\hat{x}$ computation from n samples

- Standard deviation $\sigma_{\hat{x}}^2 = \frac{1}{n}\sum_{i=1}^{n}\sigma_{x_i}^2 \Leftrightarrow \sigma_{\hat{x}} = \frac{1}{\sqrt{n}}\left(\sum_{i=1}^{n}\sigma_{x_i}^2\right)^{\frac{1}{2}}$
- Imprecision $\hat{x}_r = \frac{1}{n}\sum_{i=1}^{n}x_{i_r}$ constant influence

Comparison of the law of propagation between different types of uncertainty



Distance measurement:

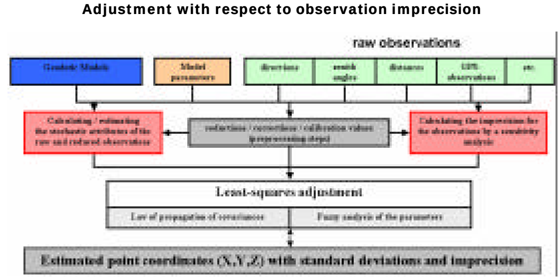
$n = 1 \ldots 20$

$s_i = 10\,\text{mm}$

$x_r = 3\,\text{mm}$

Motivation

Adjustment with respect to observation imprecision



Motivation

Tasks and methods (imprecise case)

- Determination of relevant quantities / parameters
- Calculation of observation imprecision
- Propagation of observation imprecision
- Assessment of accuracy (imprecise case)
  - ☑ Least squares adjustments
  - ☑ Statistical hypothesis tests
  - ☑ Model selection (including a sensitivity analysis)
  - ☑ Optimization of configuration
- Prototype MATLAB-Software

Propagation of imprecision

Sensitivity analysis for the calculation of observation imprecision:

- Instrumental error sources
- Uncertainties in reduction and corrections
- Rounding errors

} Influence factors (**p**)

$$\mathbf{l} = f(\mathbf{p})$$

Linearization

$d\mathbf{l} = \mathbf{F} d\mathbf{p}$

$\mathbf{F}$:= Partial derivatives for all influence factors

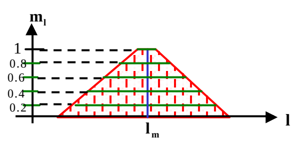
$d\mathbf{p}$:= Imprecision of the influence factors

Propagation of imprecision

Imprecise evaluation ($\mathbf{p} \in \tilde{\mathbf{p}}$):

$\mathbf{l}_m = f(\mathbf{p}_m)$ → **Stochastics (Bayesian approach)**

$\mathbf{l}_{r_a} = |\mathbf{F}|\mathbf{p}_{r_a}$ → **Observation imprecision**



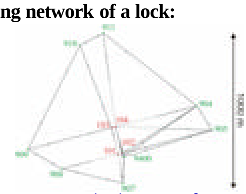
α=1.0

α-discretization

Examples

A monitoring network of a lock:

The lock Uelzen I



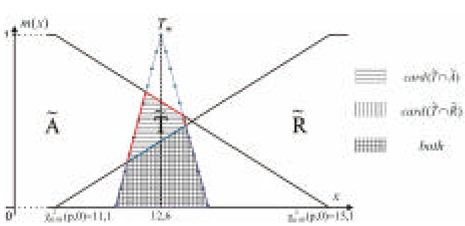
Monitoring network

Monitoring the deformations of the lock

	Distances	Zenith angles	Horizontal angles
$l_{r_a}(\alpha=0)$	0.5 mm	0.5 mgon	0.1 mgon
s	3 mm + 2 ppm	1.5 mgon	0.5 mgon

Example I

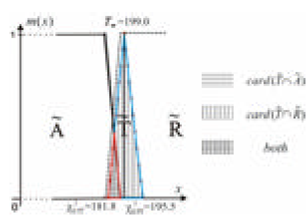
Multiple outlier test (5 horizontal directions)



Weak imprecision

Example II

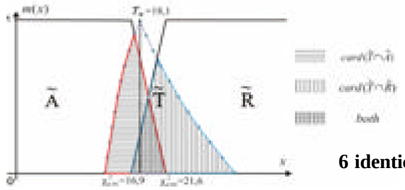
Global test in least-squares adjustment



More observations → Imprecision is becoming more important

Example III

Congruence test (1999-2004)



6 identical points

Specification	Epoch 1999	Epoch 2004
observations	317	144
parameters	60	39

Strong imprecision

Conclusions

- Geodetic deformation analysis can be extended for imprecise data
  - Based on imprecise hypothesis testing
  - 1D case is straightforward, mD case needs α-cut optimization
- The presented strategy allows in our opinion a more suitable description of measurement uncertainties than in the pure stochastic case
- Independent extension of the pure stochastic case
- Imprecision is an additive term of uncertainty what leads to a more reluctant rejection of the null hypothesis than in pure stochastic case

Future work

- Kinematic analyses (e. g., kalman filtering)
- In progress: Assessment and validation using real data to make more comprehensive statements about the so called point and system noise

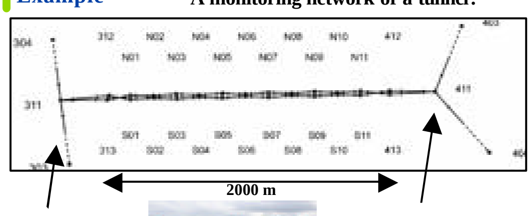
Acknowledgements

The presented results are developed within the research project KU 1250/4-1 "Geodätische Deformationsanalysen unter Verwendung von Beobachtungsimpräzision und Objektunschärfe", which is funded by the German Research Foundation (DFG). This is gratefully acknowledged.

Thank you for your attention!

Example

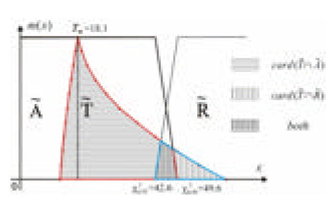
A monitoring network of a tunnel:



Reference: SBA Oldenburg

Example IV

Congruence test I [tunnel]



Very strong imprecision

16 identical points

Specification	Low tide	Tide
observations	261	261
parameters	148	148

$$\rho_{\tilde{R}}(\tilde{T}) := \min\left(\gamma_{\tilde{R}}(\tilde{T}), \delta_{\tilde{A}}(\tilde{T})\right) = 0.08$$

$$0.08 < \rho_{crit} = 0.5 \rightarrow \text{Accept } H_0$$

Propagation of imprecision (to the parameters)

precise case

(1) $\hat{\mathbf{x}} = \mathbf{x}_0 + (\mathbf{A}^T \mathbf{P}_{ll} \mathbf{A})^{+} \mathbf{A}^T \mathbf{P}_{ll} (\mathbf{l}_m - \mathbf{a}_0)$

$\underset{n,u}{\mathbf{A}}$:= Configuration matrix

$\underset{n,n}{\mathbf{P}_{ll}}$:= Weight matrix

n:= number of observations

u:= number of parameters

$\mathbf{l}$:= Vector of observations

$\mathbf{a}_0$:= Vector of approximate values of the observations

$\mathbf{x}_0$:= Vector of approximate values of the parameters

$\mathbf{y} = \mathbf{l} - \mathbf{a}_0$ Vector of reduced observations

$\mathbf{P}_{ll} = \Sigma_{ll}^{-1}$ with Σ_{ll} the variance covariance matrix (VCM) of the observations

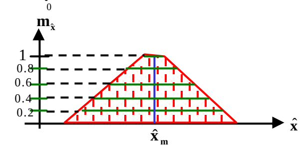
Propagation of imprecision (to the parameters)

$$\hat{\mathbf{x}}_{a,\min} = \mathbf{x}_0 + (\mathbf{A}^T\mathbf{P}\mathbf{A})^{-1}\mathbf{A}^T\mathbf{P}\left[\mathbf{l}_m - \mathbf{a}_0 - |\mathbf{F}|\left(\mathbf{p}_{r_a}\right)\right]$$

$$\hat{\mathbf{x}}_{a,\max} = \mathbf{x}_0 + (\mathbf{A}^T\mathbf{P}\mathbf{A})^{-1}\mathbf{A}^T\mathbf{P}\left[\mathbf{l}_m - \mathbf{a}_0 + |\mathbf{F}|\left(\mathbf{p}_{r_a}\right)\right]$$

imprecise case

$$m_{\hat{x}}(x) = \int_0^1 m_{\hat{x}_a}(x)\,da \quad \text{with} \quad m_{\hat{x}_a} = \left[\hat{\mathbf{x}}_{a,\min}, \hat{\mathbf{x}}_{a,\max}\right]$$



α=0....1.0

α-discretization

The interval vector $m_{\hat{x}_a}$ for the parameters is exact component for component, but overestimates the correct range of values, which is a zonotope.