Inclusion of Leveling with GNSS Observations in a Single, 3-D Geodetic Survey Network Adjustment

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Key words: Leveling, Integrated Geodesy, Geoid Accuracy, GPS-derived Orthometric Heights

SUMMARY

The current vertical datum of the United States National Spatial Reference System, known as the North American Vertical Datum of 1988 (NAVD 88), was realized by hundreds of thousands of kilometers of differential leveling between a continent-wide network of passive marks. Maintaining this large network is challenging, time-consuming, and costly, and over 25 years has passed since the completion of NAVD 88. Passive marks can be destroyed and their heights may change over time due to crustal motion, earthquakes, subsidence, and human activities such as construction and vandalism. As a result, there is a perpetual need to update and publish new heights on marks, but re-leveling is a tedious process that requires significant time and effort. To address this problem, the National Geodetic Survey (NGS) has implemented a plan to replace NAVD 88 with a new geopotential reference frame in the year 2022. This reference frame will be based on a purely gravimetric high-resolution geoid model developed from a combination of terrestrial, satellite, and recent airborne gravity measurements. As a result of this new reference frame, GNSS observations combined with the geoid model will become the primary means for deriving orthometric heights on marks. However, questions remain on what to do with the historic leveling data, as well as future leveling data collected by surveyors. Differential geodetic leveling remains much more precise than GNSS for measuring height differences between marks within a short distance of each other (e.g., less than about 5-10 km). This paper presents a case study on the inclusion of historic leveling observations with recent GNSS vectors and geoid slopes from a gravimetric geoid model to derive ellipsoid and orthometric heights on marks in a 6,400 square kilometer study area in western Oregon. A scheme was developed for weighting the leveling observations, GNSS vectors, and relative geoid height differences. The weights for the leveling data varied according to the age of the observation as well as its survey order and class per NGS survey standards. To weight the geoid height differences, a method was developed for estimating the relative accuracy of the geoid model over distances ranging from zero to about five kilometers. Afterwards, the network of observations was combined and adjusted by least squares within a 3-D geodetic model to produce the most probable geodetic coordinates on the survey marks. Combining appropriately weighted GNSS, leveling, and geoid slope observations allows GNSS and leveling observations to mitigate weaknesses in one another. For example, GNSS adds redundancy to the leveling network and helps control the increase in error when leveling over long distances. And leveling provides greater vertical precision over short distances than can be achieved with GNSS alone. The combined approach was useful for identifying marks with published leveled heights that were inaccurate or outdated, and it helped refine estimates of the most probable heights on the marks, as well as their uncertainties. By formal error propagation, network accuracies on the adjusted ellipsoid heights on leveled stations that were also observed with GNSS ranged from 0.6 to 1.2 cm (95% confidence).

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1. INTRODUCTION

For over a century, the United States' National Geodetic Survey (NGS) has established, adjusted, and maintained a continent-wide network of passive marks with published orthometric heights (i.e., *bench marks*) determined traditionally by differential leveling. The last NGS adjustment of this nationwide network of leveling observations was started in 1977 and completed in 1991 (Zilkoski et al. 1992). During that adjustment, the leveling network was minimally constrained to an adopted elevation at a tidal station named Father Point in Quebec, Canada. The result combined over 700,000 km of leveling lines and realized the datum of the current vertical component of the US National Spatial Reference System (NSRS), known as the *North American Vertical Datum of 1988* (NAVD 88).

Maintaining such a large network of bench marks is challenging, time-consuming, and costly. Over 25 years has passed since the completion of NAVD 88, and many bench marks have been disturbed or destroyed due to crustal motion, earthquakes, and subsidence, as well as due to human activities such as land development, construction, and vandalism. Heights change over time and there is a perpetual need to update orthometric heights on existing bench marks or establish heights on new marks. Re-leveling hundreds of thousands of kilometers of lines would require a significant amount of time and effort; thus, leveling is prohibitive for monitoring changes in orthometric height over time for an entire continent. Differential leveling is tedious and requires line-of-sight measurements roughly every 90 meters or less. These measurements are prone to errors which will propagate, and errors accumulate when leveling across a large continent (Roman and Weston 2011).

To address these issues, NGS implemented a plan to replace NAVD 88 with a new geopotential reference frame in the year 2022 (NGS 2013), called the *North American-Pacific Geopotential Datum of 2022* (NAPGD2022). In addition, NGS is also developing a new geometric reference frame for the year 2022 which will replace the "horizontal datum" of the NSRS, the *North American Datum of 1983* (NAD 83). NAPGD2022 will not be based on differential leveling; instead, it will be based on observations using Global Navigation Satellite Systems (GNSS) and a purely gravimetric geoid model resulting in part from the NGS "Gravity for the Redefinition of the American Vertical Datum (GRAV-D)" project (NGS 2007). With the completion of the new reference frames, a surveyor will be able to use GNSS and the geoid model to compute an orthometric height at any mark where it is suitable for making satellite observations.

Although the primary surveying technology will be GNSS, important details remain on the utility of differential leveling. For instance, differential leveling is still more precise than GNSS for determining relative height differences between marks within a short distance of each other (e.g., less than 5-10 km). For short lines or small project areas, differential leveling can be more efficient than a GNSS survey campaign. Hence, depending on the project size, surveyors will remain interested in using differential leveling, and there is a need to develop methods for integrating new leveling and GNSS observations with a geoid model in a survey network adjustment. Such an adjustment will require the use of valid control and a defensible stochastic model for all observations, and it needs to estimate most-probable ellipsoid and orthometric heights and associated uncertainties on marks in a manner that is compatible with the new reference frames.

Additionally, over the past century, a significant amount of work was completed to establish the NAVD 88 leveling network. An evaluation is needed to determine if this aging leveling network should be readjusted to fit NAPGD2022. Assuming an evaluation finds that all or part of the NAVD 88 leveling network should be readjusted, then an efficient method needs to be developed for adjusting the leveling data so that it is consistent with NAPGD2022.

1.1 Scope of paper

To address these needs, this paper focuses on testing a method for combining and simultaneously adjusting a 3-D geodetic network consisting of GNSS and leveling observations. Static GNSS observations from a 2014-2015 survey campaign on existing bench marks in western Oregon were combined with leveling observations from NGS records of historic first-, second-, and third-order leveling campaigns. A method was developed for detecting and rejecting observations that are considered outliers, and variance component estimation procedures were used for estimating realistic variances and covariances for the GNSS and leveling observations. Finally, the network of geodetic observations was simultaneously adjusted by least squares to estimate most-probable ellipsoid heights on the bench marks in the study area. The adjusted ellipsoid heights were then converted to orthometric heights using the relationship given below.

2. METHODS

2.1 Brief Background on Heights

Heights measured directly by GNSS are relative to the ellipsoid, a simple, geometric shape aimed to approximate the shape of the geoid (nominally global mean sea level). Ellipsoid heights are related to orthometric heights using the following relationship:

$$H = h - N \tag{1}$$

where H is the orthometric height measured along a plumb line from the geoid to the mark on the ground surface; h is the ellipsoid height measured along the ellipsoid normal (a line perpendicular from the ellipsoid to the mark on the ground surface), and N is the geoid height measured along the ellipsoid normal from the ellipsoid to the geoid. Technically, Eq. 1 is an approximation because the plumb line is not coincident with the ellipsoid normal and is slightly curved; however, the error is considered insignificant, likely less than 1 mm in even the most extreme settings on Earth (Jekeli 2000).

2.2 Source of Data for Analysis

Three main data types were used for this research study: reduced GNSS baseline observations (i.e., GNSS vectors), reduced differential leveling observations (i.e., differences in orthometric heights between bench marks), and geoid heights from a gravimetric geoid model.

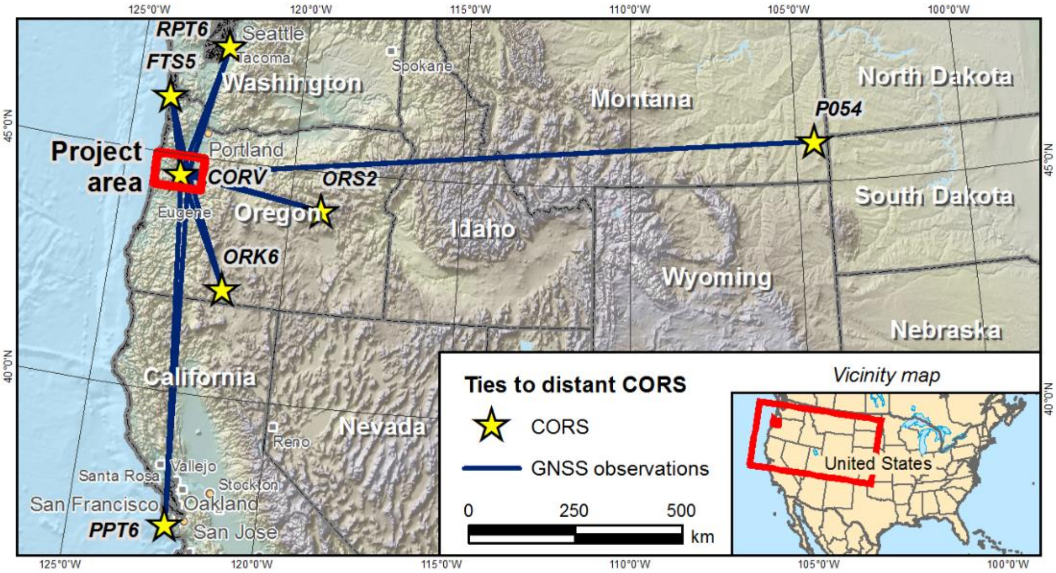
The GNSS data used for this research were from static survey campaigns conducted from 2014-2015 using dual-frequency GNSS receivers (Kerr 2015; Gillins and Eddy 2015). A total of 364 baseline observations between 40 stations were made during these campaigns, of which 207 baseline observations were between two active stations, and 157 baseline observations were from an active station to a passive mark. For the active-to-active vectors, the session duration was either 24 or 48 hours. For the active-to-passive vectors, the GNSS session duration ranged from 2.1 to 10.0 hours, with a mean of 8.1 hours. All of the baseline observations were post-processed using the NGS *OPUS-Projects* (*OP*) application (Mader et al. 2012; Weston et al. 2007) following the "OP+ADJUST" flowchart presented in Gillins and Eddy (2016). GNSS vectors to six distant continuously operating reference stations (CORS) were held as control during session baseline processing in *OP*. *OP* output the session baseline processing solutions to a "G-File", an NGS GNSS data transfer format described in detail in the NGS Bluebook (NGS, 2016). The G-file gives the GNSS vector components in delta Earth-Centered, Earth-Fixed Cartesian coordinates in a specified reference frame as well as the variance-covariance (v-c) matrix of each baseline component in the session solution.

The historic leveling observations for this research were downloaded from the NGS Integrated Databased (NGSIDB). The NGSIDB contains "reduced" observations of geopotential number differences between bench marks in an ASCII data file format known as an "R6" file. In this context, "reduced" observations means that six corrections were applied to the observed leveled differences for each leveling section, multiple runs for each section were averaged, and the corrected mean leveled differences were converted to geopotential number differences using the NAVD 88 surface gravity model (Zilkoski et al. 1992). The six corrections applied were to reduce systematic errors for rod scale, rod temperature, level collimation, atmospheric refraction, astronomic errors, and magnetic errors, as described in Balazs and Young (1982) and Holdahl et al. (1986). A total of 1278 reduced leveling observations connecting 1038 stations in the study area were downloaded from the NGSIDB. The authors then converted the leveling observations in terms of geopotential number differences to orthometric height differences.

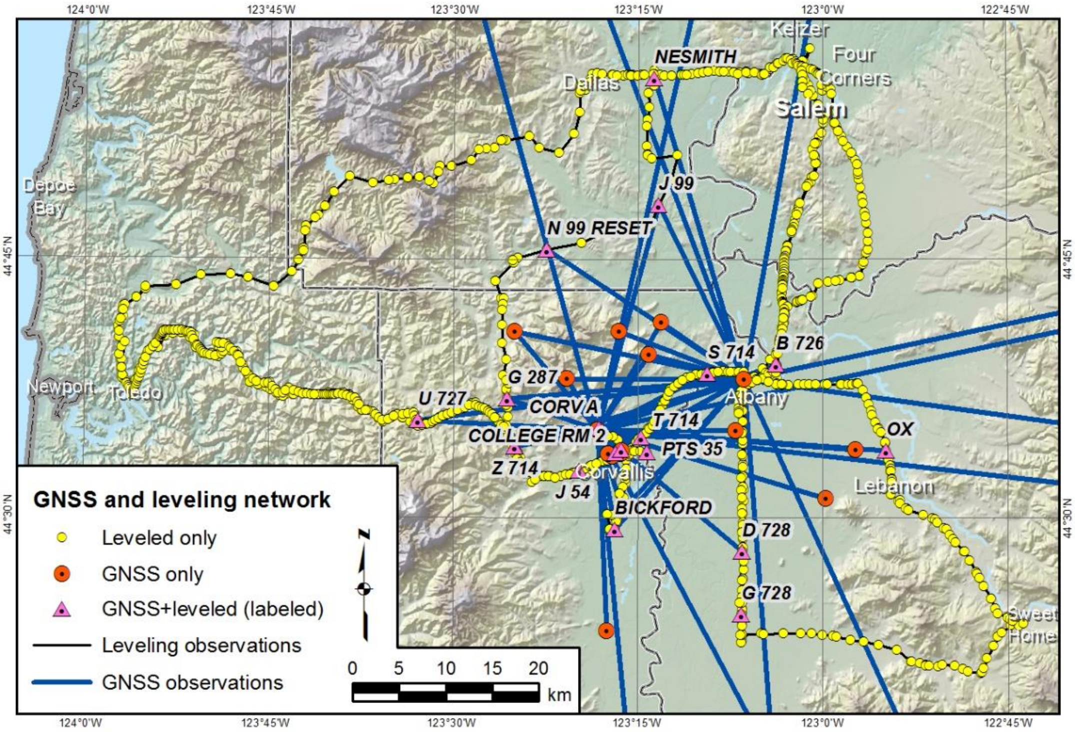
Figure 1 presents a map of the GNSS vectors and leveling observations used in the analysis. A total of 18 passive marks were both leveled and observed with GNSS. Figure 2 presents the order and class of the leveling observations, as well as the years in which the leveling was completed. The majority of the leveling met first-order class II (77.4%) or second-order class 0 (20.6%) standards (FGCS 2004). Over two-thirds (76.2%) of the leveling observations were made prior to 1945, and only 1.3% were made after the year 2000.

Geoid heights for this research were based on a gravimetric geoid model, xGEOID16B. This geoid model is the most recent "experimental" geoid model developed by NGS that includes data from the GRAV-D project. NGS currently releases annual experimental geoid models like xGEOID16B in preparation for NAPGD2022. The final geoid model for 2022 (i.e., GEOID2022) will have considerable similarity with xGEOID16B, and it is therefore advantageous to develop survey adjustment procedures based on research involving such a gravimetric geoid model.

NGS also creates hybrid geoid models derived by fitting a gravimetric geoid model by least squares collocation to GNSS observations on leveled bench marks (GPSBMs) (NGS 2015). The most recent hybrid geoid model, GEOID12B, is currently widely used in the US because it essentially provides a conversion surface from the current national geometric and vertical reference frames in the US: NAD 83 and NAVD 88, respectively. In other words, geoid heights in GEOID12B can be readily used (as per Eq. 1) for converting an ellipsoid height in NAD 83 to an orthometric height in NAVD 88.

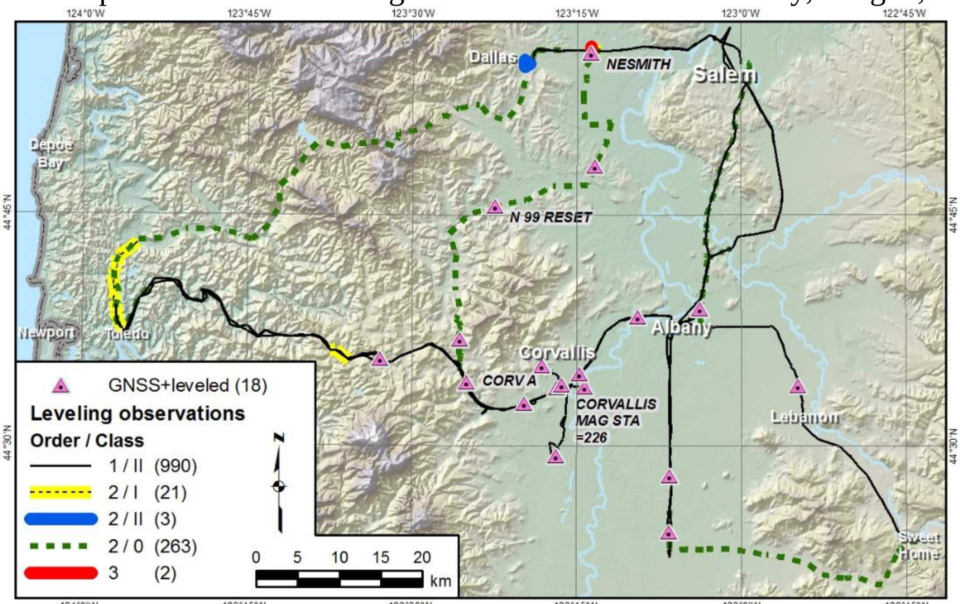


(a)

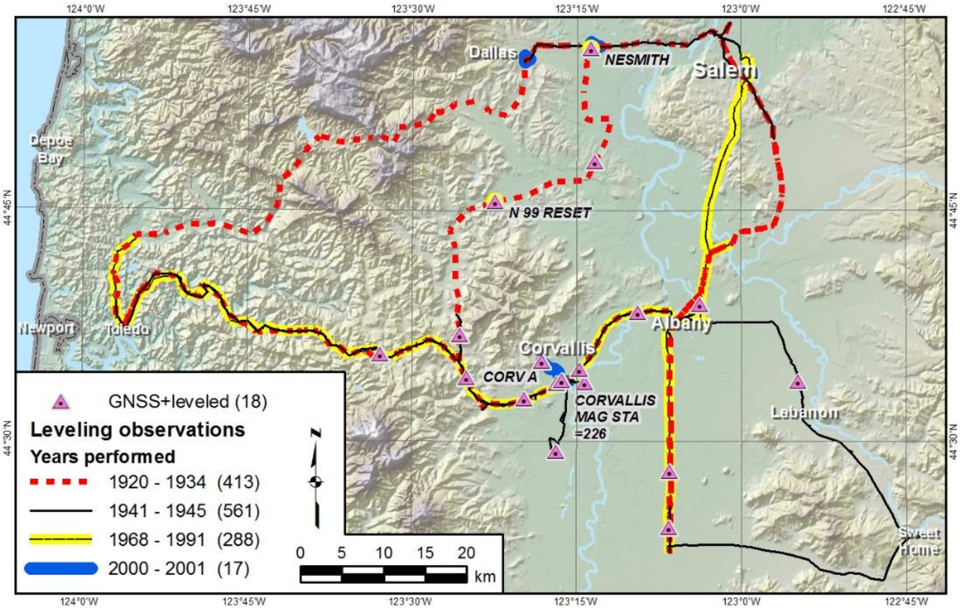


(b)

Figure 1. (a) Overview map showing GNSS vector ties to six distant CORS, and (b) project area map of GNSS and leveling networks in Willamette Valley, Oregon, USA.



(a)

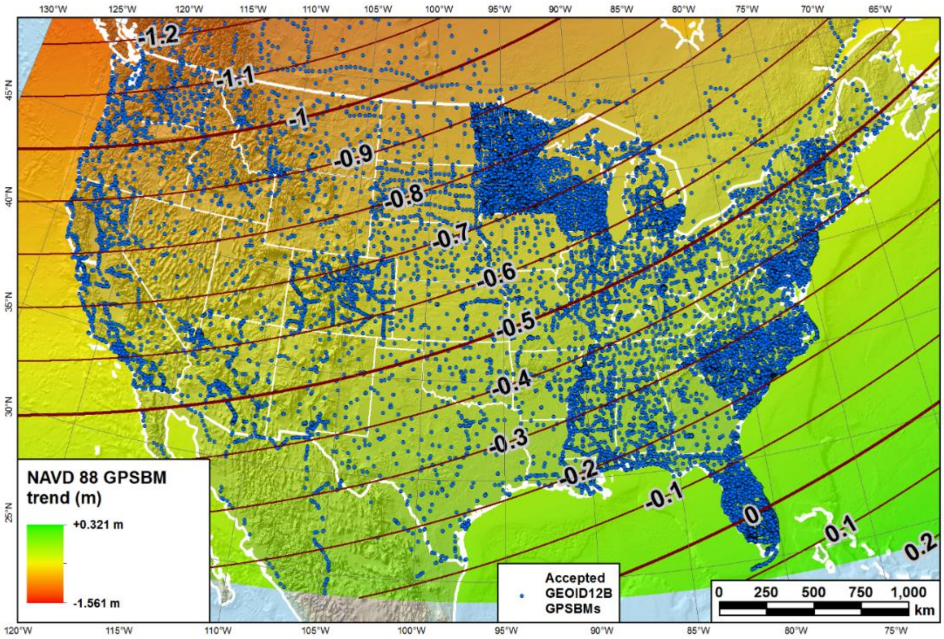


(b)

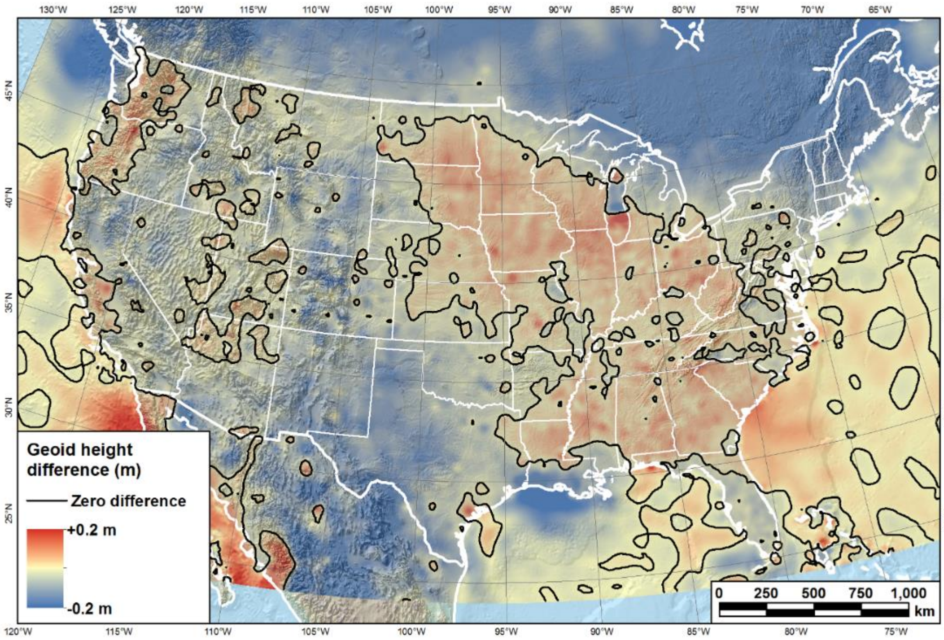
Figure 2. Distribution of leveling observations by (a) order and (b) age

It has been widely reported that NAVD 88 is systematically biased and tilted from -1.2 to +0.2 meters in the conterminous US (CONUS) with respect to satellite-derived gravity geopotential and local tide datums (Smith et al. 2013; Roman 2009; Zilkoski et al. 1992). In an effort to preserve the regional slope of xGEOID16B while accounting for the tilt and bias, a trend surface was determined by linear regression. The regression was done with the ArcGIS 10.4.1 "Trend" tool (Esri, 2016) applied to the GEOID12B GPSBM $h - H$ values minus xGEOID16B geoid heights (referenced to NAD 83) for CONUS. Figure 3a shows the resulting surface for a linear (planar) regression using the entire set of 25,196 GPSBMs used for GEOID12B, with contours in meters. Figure 3b shows the difference between the two models, as xGEOID16B (referenced to NAD 83) plus the trend depicted in Figure 3a minus GEOID12B. The only contour shown is for zero difference, which is distributed throughout CONUS.

The motivation for making xGEOID16B consistent with NAD 83 and NAVD 88 is to enable the mixed leveling and GNSS adjustments to be constrained to published control referenced to NAD 83 and NAVD 88. For all adjustments presented in this paper, geoid heights at each station in the network (Figure 1) were estimated from the tilted version of xGEOID16B referenced to NAD 83.



(a)



(b)

Figure 3. a) NAVD 88 bias and tilt based on linear regression of xGEOID16B (NAD 83) GPSBM residuals (meters); b) xGEOID16B (with NAVD 88 trend added) minus GEOID12B.

2.3 Mathematical Modeling

NGS software *ADJUST* was used for performing all of the least squares adjustments presented in this paper. *ADJUST* transforms observations to the local geodetic horizon (LGH) and performs adjustments in differential north, east, up (Δn, Δe, Δu) shifts for each component of the stations.

The specific mathematical models and observation equations used in *ADJUST* for LGH systems are given in Milbert and Kass (1987).

The general (non-linear) observation equation for all cases (whether 3-, 2-, or 1-D) is

$$\mathbf{Jx} = \mathbf{k} + \mathbf{v} \tag{2}$$

where **x** is the vector of differential corrections to the unknown parameters to be determined, **J** is the Jacobian matrix (partial derivatives of the observation equations evaluated at the computed values of the parameters), **k** is the vector of observations minus their computed values, and **v** is the vector of residuals.

The well-known least-squares solution of Eq. 2 is

$$\mathbf{x} = (\mathbf{J}^{T}\mathbf{PJ})^{-1}\,(\mathbf{J}^{T}\mathbf{Pk}) \tag{3}$$

where **P** is the weight matrix, typically estimated by taking the inverse of the v-c matrix of the unadjusted observations.

2.4 Variance Component Estimation and Rejection of Outliers

In order to develop a valid weight matrix, the variances and covariances of the leveling observations and GNSS vectors must be realistic and therefore compatible. Otherwise, either the GNSS vectors or the leveling observations may receive unrealistically high weights relative to each other and would warp the adjustment of the survey network.

In an iterative manner, standard deviations for the leveling observations were estimated, following the assumptions presented below. Afterwards, a minimally constrained least squares adjustment of only the leveling network was performed, and leveling observations with unusually high adjustment residuals were identified as outliers. For this study, if the normalized residual (i.e., vertical residual divided by its *a priori* standard deviation) was greater than 2.5, then the observation was considered an outlier, and, if warranted after inspection, it was removed from the network. After removal, a new adjustment was completed. Once all outliers were removed and when the iterated minimally constrained adjustment resulted in a standard deviation of unit weight (σ_0) equal to approximately 1, then the estimated *a priori* standard deviations of the leveling observations were considered "reasonable." Details of this adjustment process are described below, including the assumptions for estimating the *a priori* standard deviations of the leveling observations.

In order to combine and import the leveling observations and GNSS vectors in *ADJUST*, orthometric height differences from leveling were first converted to differences in ellipsoid height by the following relationship

$$\Delta h_{ij} = \Delta H_{ij} + \Delta N_{ij} \tag{4}$$

where Δh_{ij}, ΔN_{ij}, and ΔH_{ij} is the difference in ellipsoid height, geoid height, and orthometric height, respectively, from station i to station j in the leveling network. Once imported in the software, *ADJUST* transforms the observations to be in LGH coordinate space.

It is reasonable to assume leveling and gravimetric geoid height differences are uncorrelated; thus, by the special law of propagation of variance

$$\sigma_{\Delta h} = \sqrt{\sigma_{\Delta H}^2 + \sigma_{\Delta N}^2} \tag{5}$$

where $\sigma_{\Delta h}$ is the standard deviation of the leveling observation in terms of ellipsoid height differences, and $\sigma_{\Delta H}$ and $\sigma_{\Delta N}$ are the standard deviations of the differences in orthometric height and geoid height, respectively.

NGS currently uses the following relationship for estimating $\sigma_{\Delta H}$ for a leveling observation

$$\sigma_{\Delta H}(mm) = s_{\Delta L} \cdot \sqrt{d/n_r} \quad \text{(must be} \geq 0.2 \text{ mm)} \tag{6}$$

where $s_{\Delta L}$ is a scalar from Table 1, d is the shortest running distance between bench marks (km), and n_r is the number of runs between the marks used to create the reduced observation.

The scalars in Table 1 were empirically determined and used to estimate *a priori* standard deviations for the leveling observations included in the realization of NAVD 88 (Zilkoski et al. 1992). However, after removing observations with normalized residuals greater than 2.5 and performing a minimally constrained adjustment of the remaining leveling observations in terms of ΔH, σ_0 = 1.8. Thus, it appears that the scalars in Table 1 are optimistic for the leveling network in this study, and the scalars in Table 1 were thus multiplied by 1.8 and input in Eq. 6 for estimating $\sigma_{\Delta H}$.

Eq. 5 is also a function of $\sigma_{\Delta N}$. Unfortunately, an official relative geoid height error model has not been fully developed. Smith et al. (2013) presented the results of a terrestrial survey known as "GSVS11" which encompassed leveling, GNSS, deflections of the vertical, and surface gravity observations. They estimated that the error in geoid height differences was equal to 1 to 3 cm over distances from 0.4 to 325 km. Unfortunately, setting $\sigma_{\Delta N}$ to a constant value of 1 cm in Eq. 5 produces unrealistically large *a priori* values of $\sigma_{\Delta h}$. By iteration, when setting $\sigma_{\Delta N}$ to a much smaller constant value of only 2.9 mm in Eq. 5, the minimally constrained least squares adjustment of the leveling observations in terms of Δh equals 1.01. Although the statistics of the adjustment improved, it seems unrealistic for $\sigma_{\Delta N}$ to stay so small and constant over any observation distance.

Table 1. Original scalars ($s_{\Delta L}$) used for computing *a priori* **s**tandard deviations of leveling observations as a function of square root of distance (units = mm /$\sqrt{\text{km}}$).

Group	Order/Class of leveling observation (mm for 1 km distance)						
	1/0	**1/1**	**1/2**	**2/1**	**2/2**	**2/0**	**3**
Prior to 1971	0.7	1.1	2.0	2.1	2.8	3.0	4.2
From 1971 through 1978	0.7	1.1	1.4	2.1	2.8	3.0	4.2
After 1978	0.7	0.8	1.0	2.1	2.8	3.0	4.2
For Zeiss Nil level data*	—	2.0	2.0	2.8	3.0	—	—

*Almost all Nil data affected by magnetic fields were 1/1 and 1/2 obtained from 1971-1978.

Thus, it was assumed to be more realistic for $\sigma_{\Delta N}$ to follow a distance-dependent model, similar to models for $\sigma_{\Delta H}$. The following three models for $\sigma_{\Delta N}$ were tested as part of this research study:

$$\sigma_{\Delta N}\,(mm) = 2 + 1.5\ \cdot d_h \tag{7}$$

$$\sigma_{\Delta N}(mm) = 4.3\ \cdot \sqrt{d_h} \tag{8}$$

$$\sigma_{\Delta N}(mm) = 1.5\ + 2\ \cdot \sqrt{d_h} \tag{9}$$

where d_h is the horizontal distance between the bench marks in the leveling observation (km).

After removing outliers, a minimally constrained least squares adjustment of the leveling observations in terms of Δh and using estimates of $\sigma_{\Delta h}$ as a function of each of the three different error models for $\sigma_{\Delta N}$ (Eq. 5) results in σ_0 equal to approximately 1. The adjustment statistics were also similar in terms of residuals and a-posteriori standard deviations of the adjusted observations.

It can be concluded that all three models for $\sigma_{\Delta N}$ performed similarly; however, the third model (Eq. 9) was selected and used for all further adjustments presented in this paper. This choice was because at d_h> 5.4 km, $\sigma_{\Delta N}$ will exceed 1 cm according to Eqs. 7-8. Such rapid error accumulation is incompatible with the findings in Smith et al. (2013) which reported that the relative geoid height error is 1 to 3 cm at a distance up to 325 km. Per Eq. 9, $\sigma_{\Delta N}$ will exceed 1 cm at d_h> 18.1 km and will only reach 3.7 cm at d_h= 325 km.

Figure 4 presents the final *a priori* values of $\sigma_{\Delta H}$ (i.e., from Eq. 6 where the scalars in Table 1 were multiplied by 1.8) and $\sigma_{\Delta h}$(i.e., from Eq. 5 where $\sigma_{\Delta N}$ was estimated from Eq. 9) for every leveling observation in the network. As shown, the longest distance for a leveling observation in the network was only 4.3 km. After rejecting 23 of the 1278 (1.8%) observations with normalized residuals greater than 2.5, a minimally constrained adjustment of the leveling observations with *a priori* standard deviations set to values of $\sigma_{\Delta h}$ illustrated in Figure 4 results in a network σ_0 equal to 0.998.

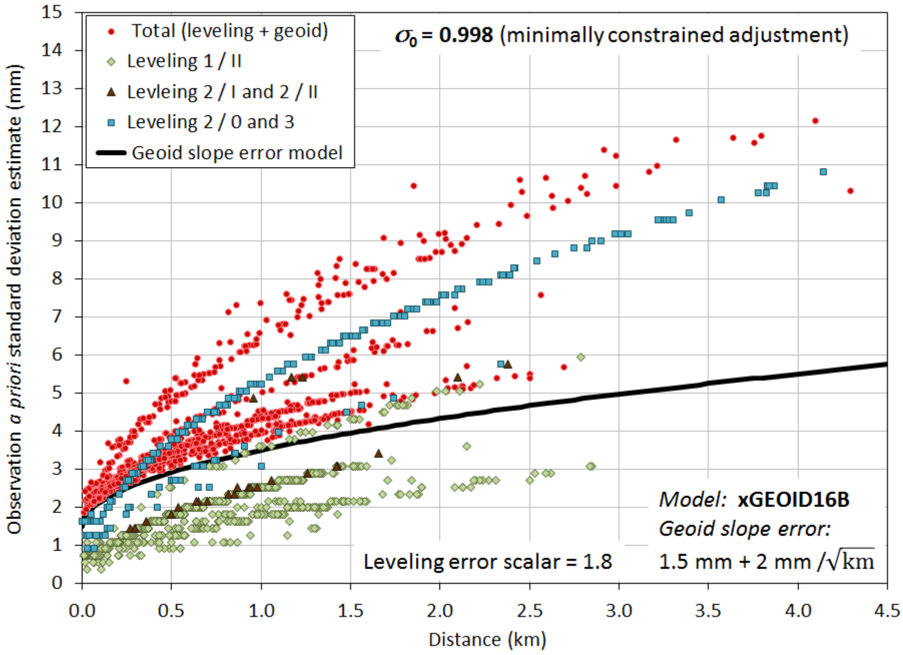


Figure 4. Final *a priori* error estimates for the leveling observations in the survey network

Similar to estimating *a priori* standard deviations for the leveling observations, estimates of realistic *a priori* variances and covariances for the GNSS vector components were also made by iteration. Using an option in *ADJUST*, the program iteratively solved for factors to scale the horizontal and vertical components of the v-c matrix output by *OP* until σ_0 of a minimally constrained least squares adjustment of only the GNSS vectors equals 1. This variance component estimation procedure is crucial for scaling the v-c matrix of the GNSS vectors because most baseline processors, including the processor in *OP*, are overly optimistic (e.g., Kashani et al. 2004; Weaver et al. 2017). *ADJUST* found a horizontal and vertical variance component equal to 18.99 and 7.70, respectively. These factors were applied to scale up the overly-optimistic v-c matrix output by *OP*.

2.5 Control and Network Adjustments

Satisfied with the removal of outliers and with the *a priori* estimates of the variances and covariances, the networks of observations were adjusted by least squares. Holding the published ellipsoid height of station S714 fixed, minimally constrained adjustments of only the leveling network, only the GNSS network, and the combined GNSS and leveling network were performed. Afterwards, the published geodetic coordinates at 18 stations (i.e., 7 CORS, 2 passive marks observed with only GNSS, and 9 passive marks in both the GNSS and leveling network) were evaluated and held as control for performing fully constrained adjustments of only the GNSS network and the combined GNSS and leveling network. The 9 passive marks in both the GNSS and leveling network were also held as control for performing a fully constrained adjustment of only the leveling network. For all constrained adjustments, weights for the coordinates of the control were derived by taking the inverse of the published variances of the published coordinates in the NGSIDB.

3. ADJUSTMENT RESULTS AND DISCUSSION

Table 2 presents results of each of the network adjustments. Because of the variance component estimation methods, the minimally constrained adjustments of the leveling-only network and the GNSS-only network resulted in σ_0 =1.00. However, the first minimally constrained simultaneous adjustment of the combined leveling and GNSS network resulted in σ_0 that was statistically not equal to 1 at the 5% significance level. Upon inspection, one of the leveling observations to a mark at the end of a leveling spur (i.e., PTS 35) had a normalized residual > 6. At PTS 35, the ellipsoid height from the constrained adjustment of the GNSS-only network was 14 cm lower than the adjusted ellipsoid height from the leveling-only network. Apparently, this mark was disturbed or settled since it was leveled. The leveling observation to PTS 35 as well as one GNSS vector with an up residual magnitude greater than 3 cm were rejected. A new minimally constrained adjustment of the combined network was performed, and σ_0 = 1.006.

All of the fully constrained adjustments had $\sigma_0 \approx 1.0$ and passed the χ^2 statistical hypothesis test at the 5% level of significance. This finding provided evidence that the estimated *a priori* variances and covariances for the observations were reasonable.

Figure 5 presents the residuals in the up component for each of the fully constrained network adjustments. The residuals for the leveling-only network were roughly 4 times more precise than the residuals for the GNSS-only network. By including the leveling observations with the GNSS vectors in the network adjustment, the precision of the up residuals nearly doubled for the GNSS+leveling network.

Table 2. Summary of minimally constrained (MC) and fully constrained (FC) network adjustments, including number of observations (N) in network, up residuals (V), and σ_0.

Adjustment	N_{leveling}	N_{GNSS}	V_{leveling} (cm)	V_{GNSS} (up) (cm)	σ_0
MC, leveling only	1256	--	-0.8 to 0.8	--	0.998
MC, GNSS only	--	359	--	-3.2 to 2.7	1.000
MC, GNSS+leveling	1256	359	-2.4 to 1.2	-4.8 to 4.4	1.110
MC, GNSS+leveling*	1255	358	-1.0 to 1.1	-2.7 to 2.7	1.006
FC, leveling only	1256	--	-0.9 to 1.1	--	1.037
FC, GNSS-only	--	359	--	-3.2 to 2.7	1.013
FC, GNSS+leveling	1255	358	-0.9 to 1.1	-2.7 to 2.7	1.017

* = rejected leveling observation to spur station PTS 35 and one of six GNSS vectors to station MAG with up residual magnitude > 3 cm.

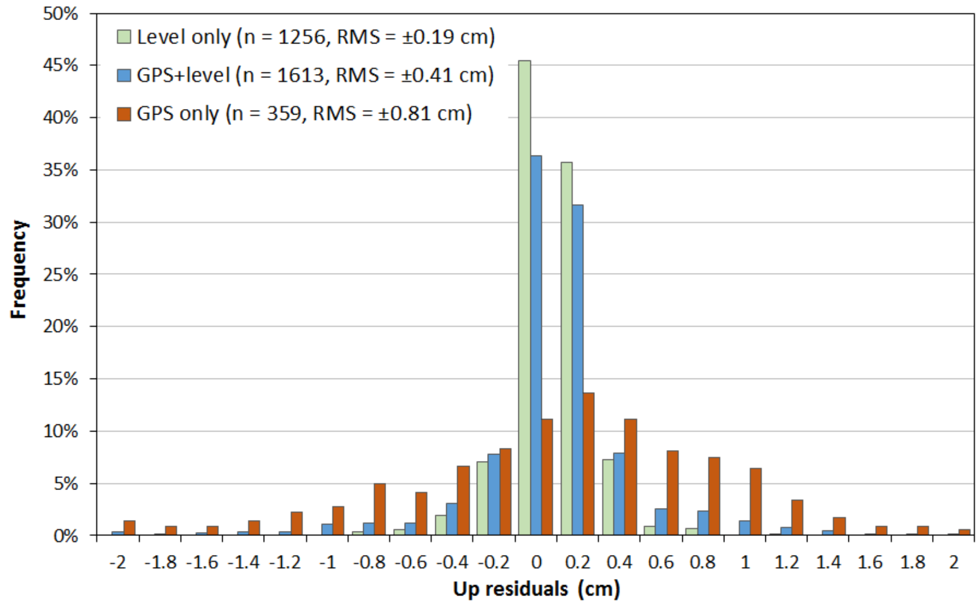


Figure 5. Up residuals for GNSS, leveling, and combined fully constrained adjustments

By formal error propagation, network and local accuracies (95% confidence) in ellipsoid height were also computed at all of the stations for each of the fully constrained adjustments per FGDC (1998) definitions. The local accuracies were only computed between stations within 100 km of each other; thus, local accuracies were not computed to the distant CORS. Figure 6 presents the median ellipsoid height accuracies for all of the stations, as well as for subsets of the stations, grouped according to whether or not they were connected to only the leveling network, only the GNSS network, or both networks.

For additional detail, Figure 7 presents ellipsoid height network and median local accuracies at 17 of 18 passive marks (PTS 35 was omitted since it was disturbed) that were both observed with GNSS and leveled. Interestingly, the network accuracies estimated from the GNSS+leveling adjustment were consistently smaller and more precise than the network accuracies from the leveling-only adjustment (Figure 7a). At some of the marks that were distant from the control stations (e.g., J99 and OX), the network accuracies from leveling-only were three times larger than from GNSS+leveling. That said, the local accuracy of leveling is superior to GNSS (Figure 7b). But, by combining the leveling observations with the GNSS vectors in the network adjustment, the resulting local accuracies were only an average of 2 mm larger than the local accuracies from the leveling-only network.

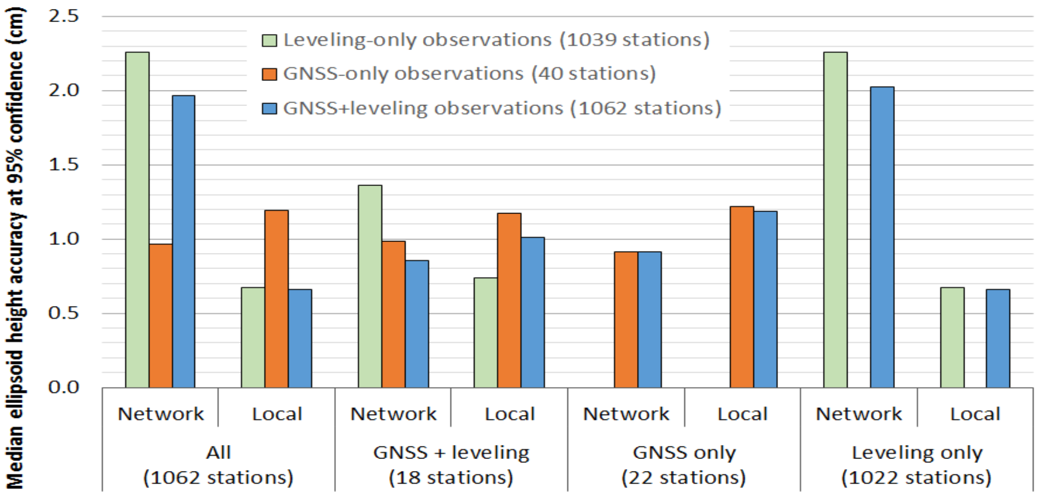
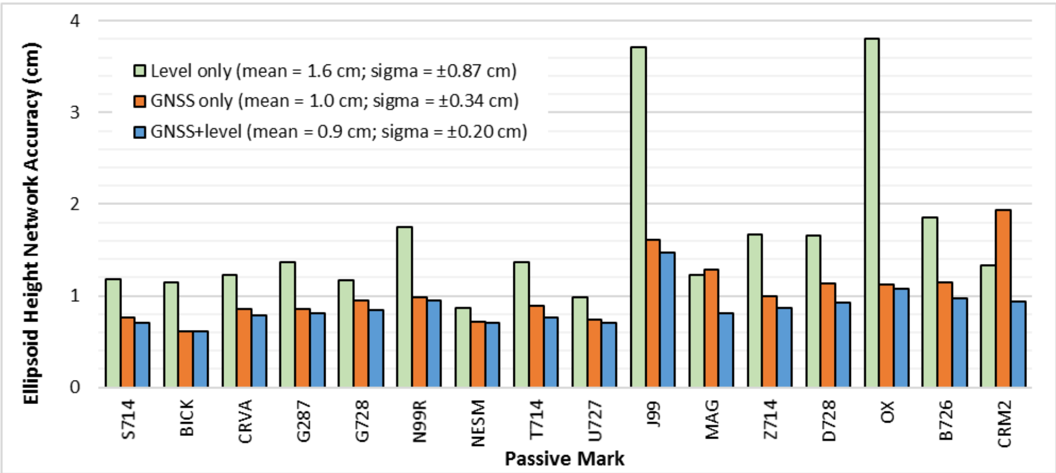
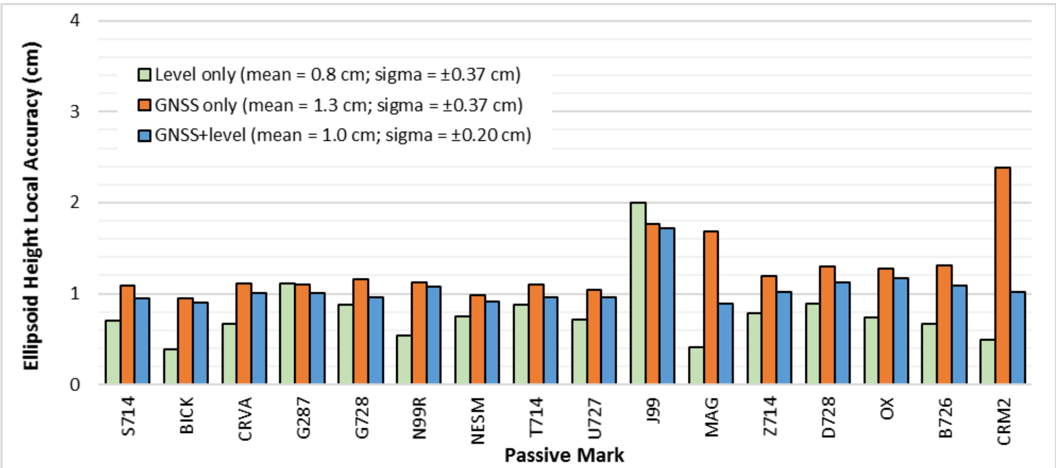


Figure 6. Median ellipsoid height network and local accuracies from fully constrained adjustments for stations grouped by connecting observation type.



(a)



(b)

Figure 7. Estimated ellipsoid height (a) network accuracies and (b) median local accuracies at passive marks in both the leveling and GNSS network based on fully constrained adjustments.

4. CONCLUSIONS

For this study, the combined GNSS and leveling network adjustment had several advantages. First, the approach was useful for identifying marks with published leveled heights that were inaccurate or outdated; for example, one mark (PTS 35) that had been disturbed since it was leveled was easily identified because its adjusted leveling residual was six times bigger than its estimated *a priori* standard deviation. Second, since leveling over short distances is highly precise, including leveling with GNSS roughly doubled the precision of the adjusted residuals in the up component in the network. Third, adding GNSS to leveling helped tie the network to the reference frame as the ellipsoid height network accuracies estimated from the GNSS+leveling network adjustment were consistently smaller and more precise than the network accuracies from the leveling-only network. Finally, although the ellipsoid height local accuracies from leveling are generally smaller than from GNSS, the estimated local accuracies from the GNSS+leveling network adjustment were only an average of 2 mm larger than from the leveling-only network. Using Eq. 1 and the tilted xGEOID16B geoid model, the adjusted ellipsoid heights found in this study can be converted to orthometric heights.

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ACKNOWLEDGEMENTS

The National Oceanic and Atmospheric Administration (NOAA) funded a portion of this research study by cooperative agreement via the Cooperative Institute for Marine Resources Studies (CIMRS), award number NA11OAR4320091. Special thanks to Drs. Michael Olsen, Christopher Parrish, and Jihye Park (Oregon State University), Mark Armstrong (NGS) and Dave Zilkoski (retired, NGS) for their advice and assistance.

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