Relative Study of the Accuracy of Spirit Leveling and GNSS Levelling

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SUMMARY

Spirit levelling is the traditional and more reliable practice in geometric geoid determination. In recent times, wider range national and regional geoid determinations are carried out with more rapid survey techniques. The rigors and time consumption of spirit levelling make it very expensive, particularly when dealing with a larger area with more number of points. On the other hand, GNSS techniques are faster and easier to operate, but are affected by such factors as signal attenuation, multipath, geometry of satellites, etc. which reduces the accuracy achievable in GNSS levelling operation irrespective of the observational method used. In this study, the least squares method was applied in calculating the error range in both observational methods as well as their accuracy level. After propagation of errors within the adjusted observations from both techniques, the spirit levelling was found to have better observational accuracy with standard deviation ranging from ±0.0001m – ±0.0075m and propagated error ranging between 0.0001m – 0.0004m within the study area. The GNSS/Leveling is also able to produce observed height to about ±0.4m residual from the spirit leveled heights with standard deviation ranging from ±0.003m – ±0.382m and propagated error ranging between 0.00065m – 0.03027m. The inference of this study however indicate that though greater accuracy is obtainable from spirit levelling operation, the GNSS levelling technique also provides reliable range of accuracies for height determination of non-geodetic ramifications.

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1. INTRODUCTION

The conventional spirit leveling technique and its variants (barometric and trigonometric leveling) have served the engineering needs of mankind for several years being the basic methods used for height determination by the early surveyors (Vanicek et al, 1980; Odumosu et al, 2016). However, the stressful procedure associated with the field observation as well as the error prone nature of the final computation in spirit leveling makes the technique a rather laborious and time consuming one. The advent of GNSS positioning on the other hand has revolutionalised leveling exercise. Modern day surveyors simply take advantage of the 3D positioning capability of satellite positioning techniques to obtain instantaneous height information (Blewitt, 1997). Although, spirit leveling results in the determination of orthometric heights the GNSS leveling results in the determination of ellipsoidal heights.

Several researches has been done to validate the suitability of the interchangeable use of these two height systems especially for engineering purposes(Olaleye et al, ; Nnam et al, 2015), but scientific enquiry of the observational accuracies in the observational techniques yielding both height systems has not been investigated using the method of propagation of errors of the ordinary least squares.

2. ERROR PROPAGATION IN LEAST SQUARES

The ordinary least squares method of data adjustment is a statistically robust method of determining best fit parameters as well as standard error of observations and parameters. This it does by minimizing the sum of squares of weighted residuals (Okwashi and Asuquo, 2012). Given the conventional least squares observation equation parameter estimation formula (equ 1) by Ayeni (2001)

$$\hat{X} = (A^TPA)^{-1}A^TPL^b \quad \text{Equ. 1}$$

where $\hat{X}^a = \hat{X}$ (vector of adjusted parameters)

A = Design Matrix

P = Weight Matrix

L^b = observation

From the law of propagation of errors (Ayeni, 2001; Ghilani and wolf, 2006), the covariance matrix of adjusted observations is given by equ 2

$\Sigma_L{}^a = A\Sigma_X{}^a A^T$ Equ. 2

Where:

A = Design Matrix

$\sum_{\hat{x}^a} = \hat{\sigma}_0^2 (A^T PA)^{-1}$ (Covariance matrix of adjusted parameters)

$Q_{xx} = A^T PA^{-1}$

Q_{xx} = Cofactor Matrix

$\sigma_{obs} = \sigma_0\sqrt{Q_{xx}}$

$$\hat{\sigma}_0^2 = \frac{\hat{V}^T P\hat{V}}{n - m}$$

$\hat{V}$ = observational residual

n = number of equations

m = number of parameters

The typical leveling route network adjustment observation equation model was used to generate the design matrix after which all other parameters were determined. The observation equation formed were 28 for each station as exemplified in equ. 3

$$\left.\begin{array}{l} CH1 = BM1 + dH1 + V1 \\ CH2 = CH1 + dH2 + V2 \\ \ldots \\ . \\ . \\ CH28 = Ch27 + dH28 + V28 \end{array}\right\} \quad \text{equ. 1}$$

The weight matrix was however constituted based on the squares of the distance of each leveling point from the starting benchmark.

3. DATA USED

Twenty eight leveled points along a profile were used for this study. The leveled points covered a distance of about 1km and all standard procedures for eliminating systematic errors and blunders in the observation were followed; since the error propagation and generally the least squares formulae works based on the assumption that only random errors do exist in the dataset analyzed (Ghilani and Wolf, 2006).

The points were located between two standard benchmarks of known orthometric height. The standard benchmarks used were MS28 and SS1MS28. The height of all points were then determined using both the spirit leveling and GNSS leveling technique. Brief description of methods used in both techniques is as given in later sections:

3.1. Spirit Leveling

The conventional two peg test and equivalence of difference of starting and closing elevation with difference of sum of sights methods were performed prior and post field observation to ensure suitability of results and complete elimination of systematic errors. All observations were ensured to have been taken with the level instrument set mid way between back and foresight. The network observation began on a known benchmark and was closed on another known benchmark.

Two lines of spirit leveling were performed along the same route, the first line was observed from the MS28 to SS1MS28 while a closing line was again observed from SS1MS

3.2. GNSS Leveling

The static method of observation was employed during the GNSS observations. By static observation, each point was occupied for 25minutes. During the observation, all positioning specifications such as the minimum GDOP, VDOP and HDOP were strictly adhered to. Also error due to multipath was avoided by ensuring that all observations were conducted in open areas with minimal signal interference with buildings and other structures.

4. RESULTS AND DISCUSSIONS

4.1. Spirit leveling height reduction and adjustment

The forward and reverse line of the spirit leveling operation was computed using the conventional height of instrument method. The mean of results obtained from the height of instrument method computation in both forward and reverse lines of leveling are as presented in table 2. As a check towards ensuring the absence of systematic errors, standard computational checks were employed as summarized in table 1 below:

Table 1: Check on Leveling computation

Forward leveling line	**Reverse leveling line**
Sum of back sight = 6.5811	Sum of back sight = 11.2975
Sum of fore sight = 9.14141	Sum of fore sight = 8.7372
Difference = -2.56031	Difference = 2.5603

The difference between the starting and closing benchmarks is 2.5603 therefore ascertaining the correctness in observation and computation of the leveling observations. The adopted values of the benchmarks are the known orthometric heights of the points therefore all the heights computed for the 28 stations are orthometric heights.

These computed heights were thereafter adjusted using the ordinary least squares technique as earlier specified in section 2. The mean spirit leveled heights, mean adjusted height of each station, mean observational standard error, mean error propagation of each observation, mean standard deviation of unit weight and differences between spirit leveled and adjusted heights of all stations was computed for both leveling lines and the results obtained (mean of forward and reverse line) is as presented in table 2:

Table 2: Summary of adjustment of spirit level heights

Sta_ID	Mean spirit level Ht(m)	**Least squares adj Results**				Residual (Level Ht - Adj Ht)
		Mean Adj Ht(m) _spirit Level	Std. dev(m)	Error (m)	Std. dev_unit weight (m)	
CH 1	189.6696	189.669	0.0001	0.00009	0.00000234	0.0006
CH 2	188.9810	188.980	0.0002	0.00010		0.0006
CH 3	188.7697	188.769	0.0003	0.00013		0.0005
CH 4	188.86974	188.869	0.0005	0.00015		0.0005
CH 5	188.9698	188.969	0.0006	0.00016		0.0004
CH 6	189.0699	189.069	0.0008	0.00018		0.0004
CH 7	189.1699	189.169	0.0010	0.00019		0.0006
CH 8	189.26998	189.270	0.0012	0.00021		0.0004
CH 9	189.37004	189.370	0.0014	0.00022		0.0003
CH 10	189.4701	189.470	0.0017	0.00023		0.0003
CH 11	189.5702	189.570	0.0019	0.00025		0.0003
CH 12	189.67022	189.670	0.0022	0.00026		0.0002
CH 13	189.77028	189.770	0.0024	0.00028		0.0002
CH 14	189.8703	189.870	0.0027	0.00028		0.0002
CH 15	189.8756	189.875	0.0030	0.00029		0.0001
CH 16	189.9345	189.934	0.0033	0.00030		0.0001
CH 17	189.9704	189.970	0.0036	0.00028		0.0002
CH 18	190.1270	190.127	0.0039	0.00031		0.0000
CH 19	190.3450	190.345	0.0042	0.00035		0.0001
CH 20	190.4267	190.427	0.0046	0.00033		0.0001
CH 21	190.5346	190.535	0.0049	0.00030		0.0001
CH 22	190.5612	190.561	0.0052	0.00038		0.0001
CH 23	190.7214	190.721	0.0056	0.00035		0.000050
CH 24	190.8645	190.864	0.0059	0.00033		0.000033

CH 25	191.2118	191.212	0.0062	0.00031		0.000008
CH 26	191.4592	191.459	0.0067	0.00044		0.000033
CH 27	191.6065	191.607	0.0071	0.00042		0.000008
CH 28	191.9539	191.954	0.0075	0.00039		-0.000017

4.2. GNSS Leveling orthometric height determination

An empirical geoid model of the study area was used to convert the GNSS/Leveling determined heights into their orthometric equivalent. The determined orthometric heights were then subjected to similar adjustment exercise as the spirit leveled elevation differences and the obtained results are as presented in table 3:

Table 3: Summary of adjustment of GNSS/Leveling derived heights

Sta_ID	Mean spirit level Ht(m)	**Least squares adj Results**				Residual (Level Ht - Adj Ht)	
		Adj GNSS derived Ht(m) _spirit Level	Std. dev(m)	Error (m)	Std. dev_unit weight (m)		Ellipsoidal Ht (m)
CH 1	189.6696	189.857	0.003	0.00065	0.0004036	-0.1874	211.6116
CH 2	188.9810	189.095	0.013	0.00153		-0.1140	211.3496
CH 3	188.7697	189.150	0.025	0.00270		-0.3799	211.8042
CH 4	188.86974	188.841	0.042	0.00408		0.0292	211.1082
CH 5	188.9698	188.601	0.059	0.00525		0.3684	211.2560
CH 6	189.0699	189.070	0.071	0.00630		-0.0005	211.1250
CH 7	189.1699	189.069	0.085	0.00630		0.1008	211.0238
CH 8	189.26998	189.672	0.099	0.00856		-0.4024	210.9271
CH 9	189.37004	189.596	0.113	0.00969		-0.2255	210.8503
CH 10	189.4701	189.445	0.127	0.01082		0.0247	210.7002
CH 11	189.5702	189.374	0.141	0.01199		0.1965	210.6285
CH 12	189.67022	189.266	0.156	0.01316		0.4038	210.5213
CH 13	189.77028	189.457	0.170	0.01429		0.3137	210.4115
CH 14	189.8703	189.835	0.184	0.01542		0.0353	210.3385
CH 15	189.8756	189.971	0.198	0.01655		-0.0949	210.2255
CH 16	189.9345	189.831	0.212	0.01768		0.1037	210.1363
CH 17	189.9704	189.737	0.226	0.01881		0.2332	209.9923
CH 18	190.1270	189.841	0.240	0.01998		0.2860	209.9392
CH 19	190.3450	190.716	0.255	0.02115		-0.3705	209.9706
CH 20	190.4267	190.265	0.269	0.02228		0.1617	211.2201
CH 21	190.5346	190.440	0.283	0.02341		0.0944	211.6953
CH 22	190.5612	190.536	0.297	0.02454		0.0250	211.7913
CH 23	190.7214	190.725	0.311	0.02567		-0.0034	211.9799

CH 24	190.8645	190.861	0.325	0.02680		0.0038	212.1158
CH 25	191.2118	191.198	0.339	0.02797		0.0135	212.4534
CH 26	191.4592	191.473	0.354	0.02914		-0.0134	212.7278
CH 27	191.6065	191.637	0.368	0.03027		-0.0305	212.8921
CH 28	191.9539	191.954	0.382	0.01542		0.0004	213.2086

5. DISCUSSION OF RESULTS

Tables 2 and 3 indicate significant differences between the spirit-leveled adjusted heights and the GNSS/Leveling derived heights. Table 4 shows a summary of statistics of the adjustment of the results from both observational techniques; while tables 5 and 6 show the stations with the strongest and weakest standard deviations and error propagation levels in both techniques. The non correspondence of the maximum and minimum value station in both techniques shows the random and independent nature of the error in both observations.

It is however obvious as shown in figures 1, 2 and 3 that the spirit leveling heights generally have lower observational standard and propagated error than the GNSS/Leveling observations. The results therefore show that although the spirit leveling provides better observational accuracy with standard deviation ranging from $\pm$0.0001m – $\pm$0.0075m and propagated error ranging between 0.0001m – 0.0004m within the study area, the GNSS/Leveling is also able to produce observed height to about $\pm$0.4m residual from the spirit leveled heights with standard deviation ranging from $\pm$0.003m – $\pm$0.382m and propagated error ranging between 0.00065m – 0.03027m.

Figure 4 also shows that although similarity exists in the pattern of rise and fall between the orthometric and ellipsoidal height that does not justify the replacement of orthometric height with its ellipsoidal equivalent as occasional pattern deviations are noticed at some stations.

Table 4: Stations with Maximum and minimum observational error in both techniques

	Summary of observed error values							
	Maximum error				Minimum error			
	Sta ID	Obs Residual	Std dev	Prop. Error	Sta_ID	Obs Residual	Std dev	Prop. Error
Spirit Lev	CH1	0.0006	0.0001	0.00065	CH25	0.000008	0.0071	0.0004
GNSS/Lev	CH12	0.4038	0.156	0.013	CH8	-0.4024	0.099	0.00969

Table 5: stations with maximum and minimum observational standard deviation

	Sta_ID	max. Std dev	min. Std dev
Spirit Lev	CH1	0.0075	0.0001
GNSS/Lev	CH12	0.382	0.003

Table 6: stations with maximum and minimum propagated error

	Sta_ID	min. Pro error	Sta_ID	max. Pro error
Spirit Lev	CH1	0.0001	CH27	0.0004
GNSS/Lev	CH1	0.00065	CH27	0.03027

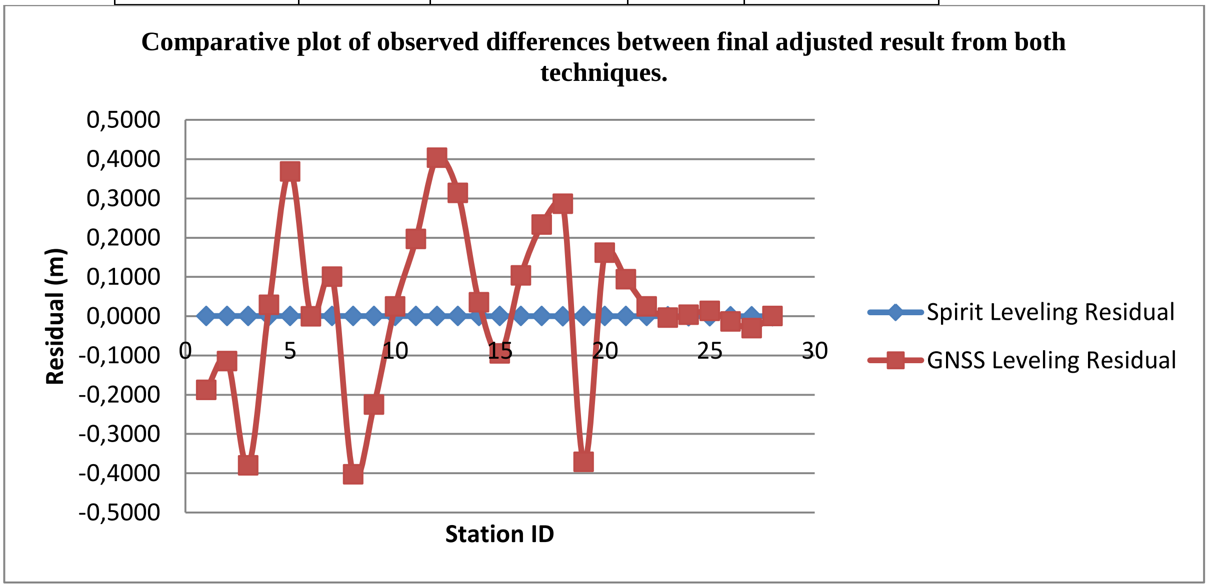

Figure 1: plot of residuals of adjusted spirit leveled heights and adjusted GNSS/Leveling heights

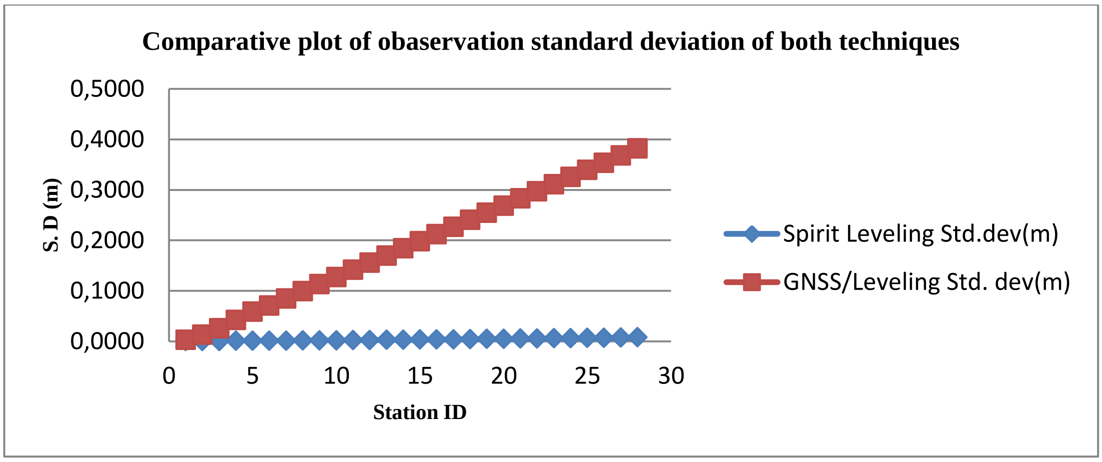

Figure 2: plot of values of standard deviation at stations from both techniques.

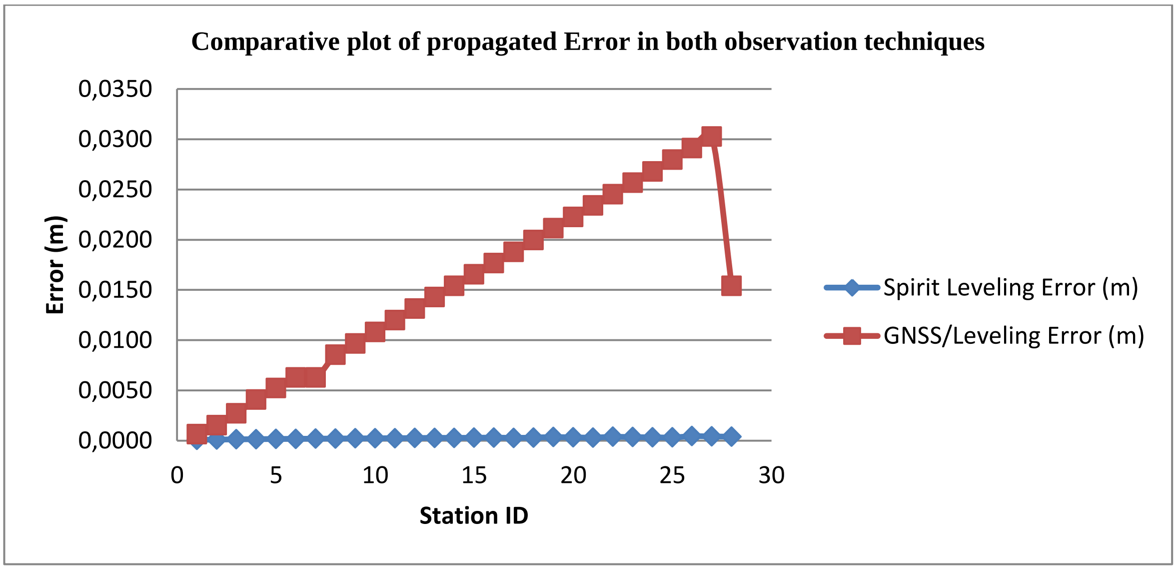

Figure 3: plot of values of propagated error from both techniques

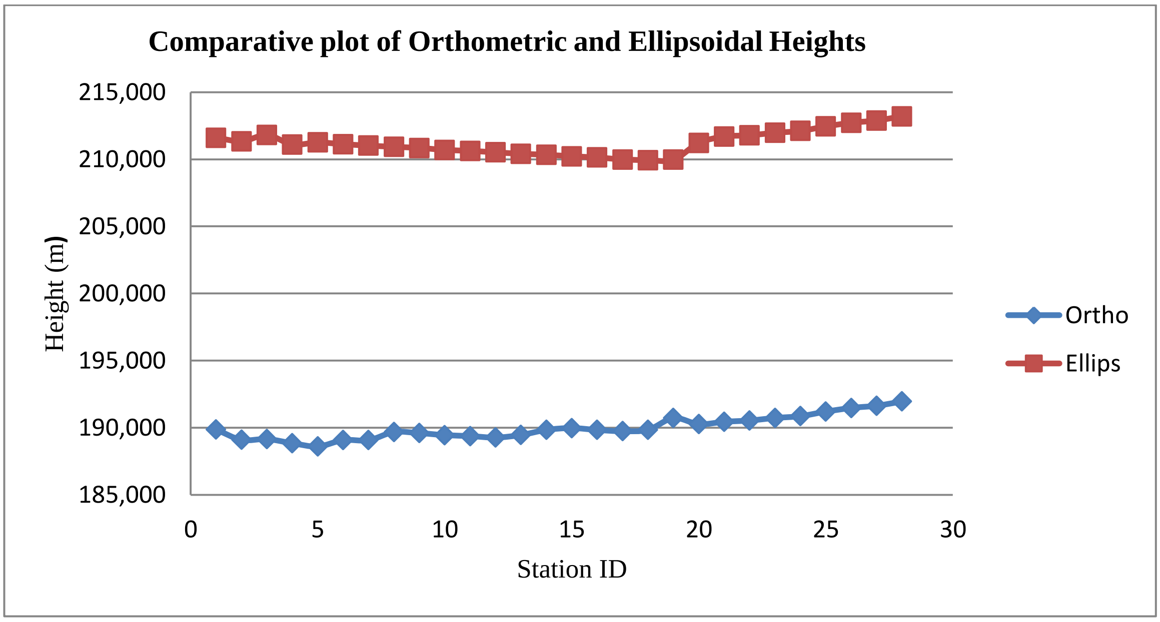

Figure 4: Orthometric and ellipsoidal heights pattern along the observed profile.

6. CONCLUSION

This experiment has identified the spirit leveling technique as the more accurate observational technique of leveling although, the GNSS/Leveling method of observation was similarly found to yield observations with third order accuracy standard errors. This method of leveling is therefore suggested as a suitable alternative in engineering and other non-geodetic surveys for quick height determination once a suitable geoid model of the area is available. The study similarly disallows the replacement of orthometric heights with ellipsoidal heights. It is suggested that increased occupation time during observations might increase the accuracy obtained in the GNSS/Leveling operation.

REFERENCES

Ayeni O.O. (2001). Adjustment Computation Manual for post graduate students in geodesy. Lecture notes of the department of Surveying and Geoinformatics, University of Lagos.

Blewitt, G. (1997). Basics of the GPS Technique: Observation Equations. In Geodetic Applications of GPS, p. 10-54, ed. B. Johnson, Nordic Geodetic Commission, Sweden, ISSN 0280-5731.

Ghilani, G. D and Wolf, P.R. (2006). Adjustment Computations; Spatial data analysis. 4th edition, John Wiley & Sons Inc. Hoboken, New Jersey.

Odumosu, J. O, Kevin, K, Omogunloye, O. G, Quadri, A, Adeleke, O. O, Olaniyi, A (2016). Empirical Geoid modeling using classical gravimetric method. Proceedings of the FIG Working week 2016 "Recovery from disaster" held at Christchurch, New Zealand on 2nd – 6th May, 2016.

Olaleye, J. B., K. F. Aleem, J. O. Olusina and O. E. Abiodun (2010). Establishment of an Empirical Geoid Model for a Small Geographic Area: A Case Study of Port Harcourt, Nigeria. Surveying and Land Information Science. 70(1): 39 - 48(10)
http://www.ingentaconnect.com/content/nsps/salis/2010/ 00000070/00000001/art00006

Okeke F. I and Nnam V. C. (2016). Determination of Best Fitting Geoid for Enugu State – Gravimetric Approach. Paper 7962, Presented at the technical Session of FIG Working Week, May 2016, Christ Church, New Zealand.

Okwashi, O. and Asuquo, I. (2012). Basics of least square adjustment computation in surveying. *International Journal of Science and Research*. Vol 3. Issue 8. ISSN (Online): 2319-7064. Pgs 1988 – 1993.

Vaníček P, Castle RO, Balazs EI (1980). Geodetic leveling and its applications. *Reviews of Geophysics and Space Physics*, vol. 18, no. 2, pp. 505-524.

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